Back Paper Examination

Physics IV,
B. Math.,
January - May 2021.
Instructor: Prabuddha Chakraborty (pcphysics@gmail.com)

Duration: 3 hours.
Total points: 75

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. Consider a step potential, which is zero for $x < 0$ and is equal to a constant value V_0 (where $V_0 > 0$) for $x > 0$. Consider an incident particle from the left with wave-vector k.
   (a) For $E < V_0$, find the reflection coefficient, the transmission coefficient and show that $R + T = 1$. [7]
   (b) Repeat the same calculation for $E > V_0$. Explain why the result for reflection coefficient does not match the classical result as we take the limit $\hbar \to 0$. [8]
2. Consider a simple harmonic oscillator with mass m and angular frequency ω. At $t = 0$, it has a wavefunction given by $\psi(x) = \cos\theta\psi_1(x) + \sin\theta\psi_2(x)$, $0 \leq \theta \leq \pi$. $\psi_{1(2)}$ are the first (second) excited wavefunctions of the simple harmonic oscillator, both normalized. Find
   (a) The expected energy, the expected value of $\hat{H}^2$ ($\hat{H}$ being the Hamiltonian/Energy operator), and ΔE as a function of time. [10]
   (b) Concentrate only on the **time-dependence** of your answers above. Which (if any) of your answers will remain the same if the coefficient of $\cos\theta$ was the normalized ground state of the simple harmonic oscillator, and the coefficient of $\sin\theta$ was the normalized first excited state of the simple harmonic oscillator. Explain your answer physically. [5]
3. Imagine a particle of mass m moving in two spatial dimensions in a rectangular well potential i.e., the potential is defined as $V(x, y) = 0$ in the rectangle ($0 \leq x \leq a$, $0 \leq y \leq b$) and $V(x, y) = \infty$, otherwise. Find
   (a) Find the **normalized** ground state of the quantum mechanical system described above. [5]

(b) Suppose $b > a$. Find the **normalized** wavefunction of the first excited state. [5]

(c) Find $\langle \hat{\vec{r}} \rangle$ and $\langle |\hat{\vec{r}}|^2 \rangle$ for the first excited state you found in the previous part. Here $\hat{\vec{r}}$ is defined as $\hat{\vec{r}} = \hat{x}\mathbf{i} + \hat{y}\mathbf{j}$, i.e., it is the two dimensional vector position operator. [5]

4. A spaceship goes to a planet with a speed of $0.9c$, spends 6 months on the planet, and returns back to earth with a speed of $0.95c$. The entire trip takes 5 years for the spaceship crew.

(a) How far is the planet according to the earth observers? [5]

(b) How long did it take the crew to get to the planet from earth, according to their own measurements? [5]

(c) How long did the entire return trip take for earth observers? [5]

5. All muons in a group move towards the earth with same speed. After moving $2911m$, half of the muons survive. The half-life of muons in their rest-frame is given by $N = N_0 \exp(-t/\tau)$, where τ is the mean half-life for muons. In the rest frame of the muons, $\tau = 2.2\mu s$.

(a) Find the speed at which the muons are moving with respect to the earth. [5]

(b) What is the mean half-life of the muons in the earth-frame? [3]

(c) A spaceship is launched from earth with speed of $0.95c$. What is the mean half-life of the muons in the spaceship frame? [7]